Half-BPS giants, free fermions and microstates of superstars

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ABSTRACT: We consider 1/2-BPS states in AdS/CFT. Using the matrix model description of chiral primaries explicit mappings among configurations of fermions, giant gravitons and the dual-giant gravitons are obtained. These maps lead to a 'duality' between the giant and the dual-giant configurations which is the reflection of particle-hole duality of the fermion picture. These dualities give rise to some interesting consequences which we study. We then calculate the degeneracy of 1/2-BPS states both from the CFT and string theory and show that they match. The asymptotic degeneracy grows exponentially with the conformal dimension. We propose that the five-dimensional single charge 'superstar' geometry should carry this density of states. An appropriate stretched horizon can be placed in this geometry and the entropy predicted by the CFT and the string theory microstate counting can be reproduced by the Bekenstein-Hawking formula up to a numerical coefficient. Similar M-theory examples are also considered.

KEYWORDS: AdS-CFT Correspondence, D-branes, Black Holes in String Theory.

Contents

1. Introduction

Dualities in string theory have proved to be powerful tools in our understanding of its physics. The most studied of these is the AdS/CFT correspondence [1] (see [2] for a review and references) which states that the $\mathcal{N} = 4$, d=4, SU(N) SYM CFT is equivalent to the type-IIB string theory on $AdS_5 \times S^5$. This being a strong-weak duality to test it one usually relies on some non-renormalisation theorems. In this context the chiral primary operators of the CFT, corresponding to BPS states on $S^3 \times R$ via the state-operator correspondence, played a very important role as their conformal dimensions are protected from quantum corrections. Under the AdS/CFT correspondence these states are dual to BPS states in the type-IIB string theory on $AdS_5 \times S^5$. It is well known however that the 1/2-BPS chiral primary operators have many possible dual descriptions on the string theory side. For small values of R-charge they are dual to multiparticle supergravity/closed string states. As the R-charge increases to $J \sim N$ the point like states are no longer good descriptions and

they are better described by large D3-branes with the same R-charges. These D3-branes come in two classes: the giant gravitons [3] and the dual-giant gravitons [4, 5]. The giant gravitons are central to understanding the 'stringy exclusion principle' from the AdS point of view [3]. For $J \sim N^2$ new geometries arise [6].

Recently it has emerged that the correlators of 1/2-BPS chiral primaries in the CFT and the physics of the corresponding dual objects in the bulk are captured by a hermitean matrix model with a harmonic oscillator potential [7, 8, 6]. In terms of the solution of this matrix model each chiral primary operator corresponds to a quantum mechanical configuration of N fermions λ_i in the single particle spectrum of a harmonic oscillator.

In this paper, to start with, we use the matrix model description to set up a one-to-one correspondence between configurations of giant gravitons and the states of N fermions in a Harmonic oscillator and similarly between configurations of dual-giant gravitons and states of the N fermion system. This gives rise to a dualtiy between configurations of giants and dual-giants. The correspondence is a consequence of particle-hole duality of the fermion picture. We also explain how stringy exclusion principle manifests itself in terms of the dual-giants. Just as there is an upper bound on the angular momentum of a single giant, it turns out that there is an upper bound, namely N, on the number of dual-giants. This result agrees nicely with the fermion picture and the duality between giant and dual-giant configurations. We study some of the interesting consequences of this novel picture.

Next we find the partition function and the asymptotic density of the 1/2-BPS states from both the CFT and the string theory. On the CFT side this amounts to counting the fixed energy configurations of N fermions in the single particle harmonic oscillator spectrum. On the string theory side we count the giant or dual-giant graviton configurations which agrees with the CFT result. We show that this density of states increases exponentially with the total energy in the large-N limit.

An exponential growth in the number of 1/2-BPS states with energy prompts one to ask if there is a 1/2-BPS 'black hole' in AdS_5 which carries this density of states. Since these states have a high amount of supersymmetry (namely 16 supercharges) their degeneracies are protected from corrections as one turns on the gravitational back reaction and hence can be compared with the microstates of an appropriate black hole geometry. However there are no 1/2-BPS black hole solutions known in AdS_5 with finite horizon area. Instead we propose that this degeneracy of states should be carried by the single charge 'superstar' geometry of [9]. This geometry arises as the extremal limit of the non-supersymmetric single charge black hole [10, 11] in AdS_5. When lifted to 10-dimensional type-IIB supergravity this solution preserves 16 supercharges and admits an interpretation as the backreacted geometry of a specific configuration of giant gravitons on S^5 (and thus denoted the 'superstar' in [9]). Since it preserves 16 supersymmetries in 10 dimensions one expects that its microstates should be dual to the 1/2-BPS operators of the $\mathcal{N} = 4$, d=4 SU(N) SYM on the boundary.

The single charge superstar geometry has a null singularity. Classically such singularities can be thought of as black holes with zero size horizons. Well studied examples of such geometries with flat asymptotes arise as the extremal limits of two charge black holes in 4 and 5 dimensions which are related to the physics of D1-D5 system. Even though

classically the area of the horizon is zero, one expects big quantum corrections to the Bekenstein-Hawking formula, e.g., [12−16]. Alternatively such singularities are expected to get corrected into black holes of finite size horizon once the quantum corrections are included [17].

Motivated by the arguments advocated by Mathur et. al. (see [18] for instance) in recent times in the context of D1-D5 systems, we place a stretched horizon in the single charge superstar geometry in 5 dimensions and show that the Bekenstein-Hawking formula for the entropy reproduces the predicted answer up to a non-zero number.

A similar analysis is carried out for the null singularities that arise in 4 and 7 dimensional gauged supergravities as well. These solutions when lifted to 11-dimensional supergravity are asymptotically $AdS_4 \times S^7$ and $AdS_7 \times S^4$ respectively and preserve 16 supercharges. These again admit interpretation as the backreactions of giants in M-theory [20, 21] now made of M2 and M5 branes. We exhibit the matching of M-theory prediction for the entropy with that of the stretched horizon picture.

The rest of the paper is organized as follows. In the section 2 we review some aspects of 1/2-BPS states in AdS/CFT and the matrix model description of these states. We argue that N is the upper bound on the number of dual-giants. In section 3 we describe the above mentioned 'duality' between configurations of giants and dual-giants using the matrix model description. In section 4 we find the partition function of the 1/2-BPS states using the fermion picture in the CFT and counting the giant graviton configurations on the string theory side. We use it to calculate the asymptotic density of states in the large-N limit and show that it grows exponentially with the conformal dimension (equivalently the R-charge). section 5 contains a review of the geometry of the single charged 'superstar' and some of its features relevant for us. In section 6 we place a stretched horizon and recover the predicted entropy up to a number. section 7 contains similar results for the M-theory superstars. In section 8 we discuss some more consequences of the duality between the giant and the dual-giant configurations. We end with some concluding remarks in section 9.[1]

2. Chiral primaries and free fermions

Let us start by briefly reviewing the relevant information of the $\mathcal{N} = 4$, d=4, U(N) SYM theory which is dual to type-IIB string theory on $AdS_5 \times S^5$ background [1] (see, e.g., [2] for a review). This theory has a large number of half-BPS operators, namely, the chiral primaries. These operators belong to $(0, l, 0)$ representation of the R-symmetry group SU(4) $\sim$ SO(6). In terms of $\mathcal{N} = 1$ notation, the $\mathcal{N} = 4$ theory has three chiral multiplets and a vector multiplet. A generic chiral primary can be written as:

$$(\mathrm{tr}(\Phi^{l_1}))^{k_1}(\mathrm{tr}(\Phi^{l_2}))^{k_2} \cdots (\mathrm{tr}(\Phi^{l_m}))^{k_m} \tag{2.1}$$

where Φ is the complex scalar of one of the three chiral multiplets. The conformal dimension Δ of these operators equals their R-charge J

$$\Delta = J \tag{2.2}$$

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[1]Some of the results of sections 2 and 3 were previously obtained in [22] using different methods. We thank Iosif Bena for pointing out [22].

Supersymmetry protects the conformal dimensions of chiral primaries from receiving quantum corrections. The operator in (2.1) has

$$\Delta = \sum_{i=1}^{m} l_i k_i. \tag{2.3}$$

For finite N the operators of type (2.1) are independent only if $l_m \leq N$. The above basis in eq. (2.1) is approximately orthonormal in the large-N limit [8]. A different orthonormal basis called the 'Schur polynomial basis' was introduced in [7]. A Schur Polynomial $\chi_R(\Phi)$ is the character of the unitary group in a given irreducible representation R. Since the irreducible representations of the unitary group can be represented by Young tableau one has a Schur polynomial for each Young tableau. The total number of boxes in a Young diagram gives the total R-charge J of the corresponding chiral primary.

The correlation functions of chiral primaries in this basis have been calculated to all order in $1/N$ and at the tree level in λ, the 't Hooft coupling in [7].[2] There, a matrix model with a harmonic oscillator potential was proposed to capture the correlation functions of chiral primaries. This matrix model can be obtained as a truncation of the d=4 SYM on $S^3 \times R$ down to the zero modes over S^3 in the weak coupling and keeping just one complex scalar Φ [8]. This model was further studied in [8, 24, 6]. Some relevant features of this are briefly reviewed below.

As it has been explained in [7, 8] (see also [25]) the matrix model can be reduced to the quantum mechanics of N fermionic eigenvalues λ_i of the matrix in a harmonic oscillator potential. The hamiltonian of this system is

$$H = \sum_{i=1}^{N} \left(\lambda_i^\dagger \lambda_i + \frac{1}{2} \right). \tag{2.4}$$

Each stationary state of this quantum mechanics is given by a configuration of N-fermions in the harmonic oscillator energy spectrum $E_j = j + 1/2$, $j = 0, 1, \ldots$. Since there are N fermions the vacuum energy is $E_0 = \frac{1}{2} + \frac{3}{2} + \cdots + \frac{2N-1}{2} = \frac{N^2}{2}$.

The Hilbert space is spanned by N-particle states labeled by N single particle levels f_k or an N-vector $\vec{f} = (f_1, f_2, \ldots, f_N)$ where $0 \leq f_1 < f_2 < \ldots < f_N$. The eigenvalue of the N-particle hamiltonian (2.4) on this state is $E = \frac{N}{2} + \sum_{k=1}^{N} f_k$. We choose to measure the energies of the states to be the difference of the full energy and the ground state energy: $\Delta = E - E_0$.

Note that each excitation of the fermion system can be mapped uniquely to a U(N) Young tableau and thus a representation R of U(N). This is done by mapping the particle excitations to successive rows of the tableau. In our case since there are N fermions, the Young tableau contains a maximum of N rows. The number of boxes in the kth row is $f_k - (k-1)$. Therefore each chiral primary written in the Schur polynomial basis corresponds to a unique configuration of the fermion system [7, 8]. We will make use of this fermion picture later on.

Let us next turn to the string theory duals of chiral primaries.

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[2]See [23] for a discussion of these correlators for the gauge group SU(N). We will however continue to use gauge group U(N) as we will be interested in only large-N limits for most part.

2.1 Giants and dual-giants

As mentioned in the introduction the chiral primary operators have many possible dual descriptions on the string theory side. For small values of R-charge they are dual to supergravity modes. However as the R-charge increases $J \sim N$ the duals are better described semiclassically by D3-brane configurations with the same R-charges. Further, these D3-branes come in two classes: the giant gravitons [3] and the dual-giant gravitons [4, 5]. We will be interested in counting the duals of chiral primaries later on. So to avoid overcounting we have to restrict ourselves to either counting the supergravity KK modes or the giant D3-branes. Since we are interested in states with very large R-charge we choose the giant D3-brane basis. It turns out that we should not count giant and dual-giant configurations separately as there is a 'duality' relating both.

Let us now briefly review some relevant aspects of the giant and the dual-giant gravitons in type-IIB string theory on $AdS_5 \times S^5$. Giant gravitons [3] are D3-branes wrapping an S^3 inside the S^5 and rotating along one of the transverse directions within the S^5. They preserve 16 of the 32 supersymmetries[3] of $AdS_5 \times S^5$. Let us work with the following coordinate system for $AdS_5 \times S^5$ in global coordinates.

$$ds^2_{AdS_5} = -\left(1 + \frac{r^2}{L^2}\right) dt^2 + \frac{dr^2}{1 + \frac{r^2}{L^2}} + r^2 d\Omega_3^2, \tag{2.5}$$

$$ds^2_{S^5} = L^2 \left(d\alpha^2 + \sin^2\alpha \, d\beta^2 + \cos^2\alpha \, d\xi_1^2 + \sin^2\alpha \left[\cos^2\beta \, d\xi_2^2 + \sin^2\beta \, d\xi_3^2\right]\right). \tag{2.6}$$

Here the ranges of the coordinates are: $0 \leq r < \infty$, $0 \leq \alpha, \beta \leq \pi/2$, and $0 \leq \xi_i \leq 2\pi$ for $i = 1, 2, 3$. Further assume that the D3-brane wraps the $S^3 \subset S^5$

$$ds^2_{S^3} = L^2 \sin^2\alpha \left[d\beta^2 + \cos^2\beta \, d\xi_2^2 + \sin^2\beta \, d\xi_3^2\right] \tag{2.7}$$

and rotates along the ξ_1 direction. The angular momentum of a single such D3-brane is given by

$$P_{\xi_1} = N \sin^2\alpha. \tag{2.8}$$

Thus we have $P_{\xi_1} \leq N$ which realises the stringy exclusion principle.

On the other hand the dual-giant gravitons [4, 5] are D3-branes wrapping $S^3 \subset AdS_5$ and rotating along a maximal circle of S^5. They again preserve 16 supercharges of the background and carry D3-brane dipole moment. For a given angular momentum P_{ξ_1} the radius r at which the D3-brane stabilises is given by

$$r = L \sqrt{\frac{P_{\xi_1}}{N}} \tag{2.9}$$

where L is the radius of AdS_5. Since the radial coordinate r ranges from 0 to ∞ the dual-giants can have arbitrary (integer valued) angular momenta.

This raises the question of how the stringy exclusion principle manifests itself for the dual-giants. To answer this let us note a subtle effect which restricts the total number of

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[3]There are also giant gravitons that carry more than one R-charge which are 1/4 or 1/8 supersymmetric [26]. We will not consider these configurations here.

dual-giants that one can place in $AdS_5 \times S^5$ (see also [22]). Since a dual-giant occupies three of the four spacelike coordinates in AdS_5, it acts like a domain-wall and the flux of $F^{(5)}$ measured on either side of this domain-wall differs by one unit with the lesser value on the inside of S^3 that the dual-giant wraps [5]. So if we have m dual-giants in AdS_5 the $F^{(5)}$ flux measured inside the inner most dual-giant will be $N - m$ units. For $m = N$ the five form flux inside the innermost dual-giant vanishes. Since it is crucial to have non-zero flux to stabilise a dual-giant at a non-zero radius and to produce a geometry where there are closed orbits[4], it follows that we can not have any more dual-giants in the system. This is the manifestation of 'stringy exclusion principle' for the dual-giants.

Next we propose a one-to-one map among configurations of giants and configurations of dual-giants using the fermion picture coming from the CFT.

3. Mapping branes and fermions

In a quantum theory one expects that P_{ξ_1} is quantised. Hence the allowed values of α would be discrete such that P_{ξ_1} takes values $0, 1, 2, \ldots, N$. A general configuration of giants is then given by an N-vector $\vec{b}_1 = (r_1, r_2, \ldots, r_N)$ where the integers $r_k \in [0, \infty)$ denote the number of giant gravitons with angular momentum $P_{\xi_1} = k$. The total energy (and therefore the angular momentum) of this configuration is $\sum_{k=1}^{N} k\, r_k$.

Taking this into account a general configuration of dual-giants is also given by an N-vector $\vec{b}_2 = (s_1, s_2, \ldots, s_N)$. Here the integers s_k are such that $0 \le s_N \le \ldots \le s_1 < \infty$ and s_k denotes the angular momentum of the kth dual-giant away from the boundary of AdS_5. The total energy of this configuration is given by $H_{\vec{b}_2} = \sum_{k=1}^{N} s_k$.

To summarise, a configuration of the N fermion system is specified by an N-vector $\vec{f} = (f_1, f_2, \ldots, f_N)$ with $0 \le f_1 < f_2 \ldots f_N \le \infty$ and f_i denoting the level number of the ith fermion. The total energy of such a configuration is

$$H_{\vec{f}} = -\frac{N(N-1)}{2} + \sum_{k=1}^{N} f_k. \tag{3.1}$$

A configuration of giant gravitons is specified by another N-vector of integers $\vec{b}_1 = (r_1, r_2, \ldots, r_N)$ with $0 \le r_k \le \infty$. Each r_k denotes the number of giant gravitons at level-k in the N-level system. The energy of this configuration is

$$H_{\vec{b}_1} = \sum_{k=1}^{N} k r_k. \tag{3.2}$$

The dual-giant graviton system is described by yet another N-vector of integers $\vec{b}_2 = (s_1, s_2, \ldots, s_N)$ with $s_1 \ge s_2 \ge \ldots \ge s_N \ge 0$. Each s_k denotes the angular momentum of the kth dual-giant D3-brane away from the boundary of AdS. The energy of this system is

$$H_{\vec{b}_2} = \sum_{k=1}^{N} s_k. \tag{3.3}$$

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[4]I thank Rob Myers for suggesting this argument.

A fermion configuration can be specified either in terms holes or particles. We first observe that each hole can be associated with a giant graviton and each particle with a dual-giant graviton. For instance a hole at the level-0 (the ground state of the single particle harmonic oscillator Hilbert space) can be thought of as the one in which all the N particles are excited by one energy level each. As explained in [8], a hole at the level-0 is dual to a single giant graviton with the maximum possible angular momentum N. On the other hand in terms of dual-giants this excitation corresponds to having N dual-giants with one unit of angular momentum each. Of course the description in terms of a single giant is a better one as the probe brane approximation will be valid. Similarly consider an excitation of the topmost fermion by n levels. This, as in [8] can be thought of as a dual-giant with n units of angular momentum. But this can also be associated with a configuration of n smallest size giant gravitons. Again the former would describe the system better than the latter. Similarly one can analyse a given configuration of fermions and associate a unique giant or a dual-giant configuration with it. Doing this we find the following maps between the fermion configurations and the giant and the dual-giant configurations:

$$r_N \leftrightarrow f_1, \quad r_{N-i} \leftrightarrow f_{i+1} - f_i - 1, \quad i = 1, 2, \ldots, N-1$$
$$s_{N-i} \leftrightarrow f_{i+1} - i, \quad i = 0, 1, \ldots, N-1 \tag{3.4}$$

Clearly under these identifications the hamiltonians of the systems match along with the restrictions on the vectors $\vec{f}$, $\vec{b}_1$ and $\vec{b}_2$. Since there is a single fermion configuration for either of the two bosonic (giant and dual-giant) configurations it is natural to conjecture that the two bosonic systems should be 'dual' to each other with the following mapping:

$$s_i \leftrightarrow \sum_{k=i}^{N} r_k, \quad i = 1, 2, \ldots, N \tag{3.5}$$

This 'duality' map of (3.5) between the configurations of giants and dual-giants is a direct result of the duality between descriptions of the fermion system in terms of particles or holes.[5] Recall that each fermion configuration is uniquely determined by a Young tableau. Using (3.4) the number of boxes in each row can be identified with the corresponding s_k. Therefore a given Young tableau can be associated uniquely to a specific configuration of dual-giants with the number of boxes in the kth row giving the angular momentum of the corresponding dual-giant. The fact that there are a maximum of N rows reflects the fact that there is an upper bound on the number of dual-giants. This is the particle description of the fermion system.

On the other hand the same fermion configuration can be equivalently described by holes. And the hole excitations can also be described by the same Young tableau. This picture can be associated with a configuration of giants with each column representing a giant graviton and the number of boxes in that column being its angular momentum. This type of a duality when applied to M-theory context would mean that some configurations of giants made of M5 (M2) branes in $AdS_4 \times S^7$ ($AdS_7 \times S^4$) are dual to the corresponding

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[5]The relation in eq. (3.5) for the case of single non-vanishing r_k was proposed earlier in [22].

configurations of dual-giants made of M2 (M5) branes. A similar duality has already been proposed by [28] in the context of M-theory in a pp-wave background.

Even though there are dualities between different descriptions of chiral primaries on the string theory side, one has to keep in mind that for most of the situations only one of the three candidates, namely the point like KK modes, the giant gravitons or the dual-giant gravitons is a good description but not all. In some cases none of them alone describes the true physics in which case one has to work with the full supergravity solution [6]. We next turn to enumerating the 1/2-BPS states and finding their asymptotic degeneracies for fixed Δ (J).

4. Asymptotic densities of 1/2-BPS states

One can write down a generating function to summarise the number of independent chiral primary operators for a given Δ. One can check that the function

$$Z_N(q) = \prod_{n=1}^{N} (1 - q^n)^{-1} = \sum_{\Delta=0}^{\infty} d_\Delta q^\Delta \qquad (4.1)$$

fits the bill where $q = e^{-\beta}$ and d_Δ gives the number of independent chiral primaries with conformal dimension Δ. This partition function can be calculated in many ways. Before analysing (4.1) further for the asymptotic density of states let us derive this from the matrix model by counting the fermion configurations and from string theory by counting the configurations of giant gravitons.

4.1 Partition function from the matrix model

The partition function of the system of N fermions in a harmonic oscillator potential is [25]:

$$Z_N(q) = q^{-\frac{N^2}{2}} \operatorname{Tr} q^H = q^{-\frac{N^2}{2}} \sum_{f_1=0}^{\infty} \sum_{f_2=f_1+1}^{\infty} \cdots \sum_{f_N=f_{N-1}+1}^{\infty} q^{\sum_{n=1}^{N}(f_n+\frac{1}{2})} \qquad (4.2)$$

with $q = e^{-\beta}$. To perform the sums we make the following change of variables: $r_N = f_1$, $r_{N-i} = f_{i+1} - f_i - 1$ for $i = 1, 2, \ldots, N-1$. In terms of the new variables eq. (4.2) becomes

$$Z_N(q) = \sum_{r_1=0}^{\infty} \sum_{r_2=0}^{\infty} \cdots \sum_{r_N=0}^{\infty} q^{\sum_{j=1}^{N} j\, r_j} \qquad (4.3)$$

which can be summed easily to get

$$Z_N(e^{-\beta}) = \sum_J d_\Delta e^{-\beta\Delta} = \prod_{n=1}^{N} \frac{1}{1 - q^n}. \qquad (4.4)$$

Here $\Delta = J$ is again the total U(1) R-charge and d_Δ is the degeneracy of states with fixed Δ. So we have recovered the partition function of chiral primaries (4.1) using the matrix model.

4.2 Partition function from giants

As discussed earlier in section 2.1 there are three different known duals of 1/2-BPS states of the SYM. We choose to count the giant and the dual-giant configurations. Invoking the 'duality' of section 3 between configurations of giants and dual-giants we can simply choose to count giant graviton configurations. It is clear that counting the dual-giants will give identical results.

It is known from [29] that there are no 1/2-BPS fluctuations of the giant gravitons and therefore it is sufficient to treat these as simple particles. Thus the problem of counting the giant graviton configurations reduces to that of counting the configurations of bosons in an equally spaced N-level system. As explained earlier a general configuration of the giants is labeled by N integers $\vec{b}_1 = (r_1, r_2, \ldots, r_N)$. The total R-charge of such a configuration is:

$$P_{\xi_1} |\vec{b}_1\rangle = \left(\sum_{k=1}^{N} k r_k \right) |\vec{b}_1\rangle. \tag{4.5}$$

Thus counting the number of giant graviton configurations with a fixed total angular momentum again reduces to the problem of partitions of an integer $Q = \sum_{k=1}^{N} k r_k$. The partition function of this problem is

$$\begin{aligned} Z_N(q) &= \sum_{r_1, \ldots, r_N} q^{\sum_{k=1}^{N} k r_k} \\ &= \prod_{k=1}^{N} \left(\sum_{r_k=0}^{\infty} q^{k r_k} \right) = \prod_{k=1}^{N} \frac{1}{1 - q^k} \end{aligned} \tag{4.6}$$

where $q = e^{-\beta}$. Of course this is precisely the same partition function eq. (4.4) obtained from the CFT (matrix model) side.

4.3 The asymptotic degeneracy and the entropy

To further analyse $Z_N(q)$, define the free energy

$$\begin{aligned} F(q, N) &= \ln Z_N(q) = -\sum_{n=1}^{N} \ln(1 - e^{-\beta n}) \\ &= -\sum_{n=1}^{\infty} \ln(1 - e^{-\beta n}) + \sum_{n=N+1}^{\infty} \ln(1 - e^{-\beta n}) \\ &= \ln Z_\infty(q) + \sum_{p=1}^{\infty} \frac{1}{p} \frac{e^{-\beta(N+1)p}}{1 - e^{-\beta p}}. \end{aligned} \tag{4.7}$$

Using eq. (4.7) we can write $Z_N(q)$ as

$$\begin{aligned} Z_N(e^{-\beta}) &= \frac{e^{-\frac{\beta}{24}}}{\eta(e^{-\beta})} \exp\left[\sum_{p=1}^{\infty} \frac{1}{p} \frac{e^{-\beta(N+1)p}}{1 - e^{-\beta p}} \right] \\ &= \frac{e^{-\frac{\beta}{24}}}{\eta(e^{-\beta})} \left[1 + \mathcal{O}(e^{-\beta(N+1)}) \right] \end{aligned} \tag{4.8}$$

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where $\eta(q)$ is the standard Dedikind's eta function: $\eta(q) = q^{\frac{1}{24}}\prod_{n=1}^{\infty}(1-q^n)$. In the large-$N$ limit, we can neglect the $\mathcal{O}(e^{-\beta(N+1)})$ corrections to $Z_N(q)$ and we will do so for the rest of this paper.

From eq. (4.1) d_Δ is given by:

$$d_\Delta = \frac{1}{2\pi i}\int_C d\beta\, e^{\beta\Delta}\, Z_N(e^{-\beta}) \tag{4.9}$$

We are interested in extracting d_Δ for large-Δ and N. Substituting the leading value of $Z_N(q)$ from eq. (4.8) into eq. (4.9) we have

$$d_\Delta = \frac{1}{2\pi i}\int_C d\beta\frac{e^{\beta(\Delta-\frac{1}{24})}}{\eta(e^{-\beta})}. \tag{4.10}$$

The asymptotic density d_Δ for large Δ can be found using the standard methods in the saddle point approximation to find:

$$d_\Delta \approx \frac{1}{4\sqrt{3}\,\Delta}\, e^{\pi\sqrt{\frac{2\Delta}{3}}}. \tag{4.11}$$

Thus the density of the 1/2-BPS states grows exponentially. From eq. (4.11) the Boltzman's entropy formula $S = \ln d_\Delta$ gives

$$S = \left(\frac{2\pi^2}{3}\right)^{1/2}\sqrt{\Delta} + \cdots \tag{4.12}$$

where $\cdots$ are the corrections negligible in the large-N and large-Δ limit (with $N \gg \sqrt{\Delta}$) which we drop henceforth.

Thus we see that if there is a physical system with the 1/2-BPS states as its microstates then it is expected to have a macroscopic entropy for large Δ. So it is natural to ask whether there is a candidate in the bulk AdS_5 which carries this entropy. The dual should be asymptotically AdS_5, should preserve 16 supercharges when lifted to a 10-dimensional solution and carry just one U(1) R-charge. There are no BPS black holes with these properties and with finite size horizons in AdS_5. Instead in what follows we propose that the single charge 'superstar' of [9] should have this entropy.

5. The R-charge black holes in AdS_5 and the superstar

Let us review the relevant 'superstar' solution of $\mathcal{N} = 2$, d = 5 gauged supergravity and its 10-dimensional lift. The bosonic field content of the 5-dimensional gauged supergravity is the metric, two scalars parametrised by X_i, $i = 1, 2, 3$ satisfying $X_1 X_2 X_3 = 1$ and three abelian gauge fields A_i. The theory admits non-extremal charged black holes [10, 11]:

$$ds_5^2 = -(H_1 H_2 H_3)^{-2/3}f dt^2 + (H_1 H_2 H_3)^{1/3}(f^{-1}dr^2 + r^2 d\Omega_3^2),$$
$$X_i = H_i^{-1}(H_1 H_2 H_3)^{1/3}, \qquad A_i = (H_i^{-1} - 1)dt, \tag{5.1}$$

where

$$f = 1 - \frac{\mu}{r^2} + \frac{r^2}{L^2}H_1 H_2 H_3, \quad H_i = 1 + \frac{q_i}{r^2}. \tag{5.2}$$

Here μ is the non-extremality parameter, L is the radius of the asymptotic AdS_5 and q_i determine the independent $U(1)^3$ charges. As noted in [9], the area of the horizon shrinks as μ decreases. There is a qualitative difference between the case when only one charge is non-vanishing , e.g., $q_1 \neq 0$, $q_2 = q_3 = 0$, and the case when more than one charges are non-vanishing. If only q_1 is non-zero then the horizon shrinks to zero size precisely when $\mu \to 0$ and the solution becomes BPS with a null singularity. For the other cases the horizon shrinks to zero size for some nonzero $\mu = \mu_{crit}$ leaving behind a naked timelike singularity as $\mu \to 0$.

Since we are interested only in the extremal solution and with just one U(1) R-charge we will set $q_1 = Q$, $\mu = q_2 = q_3 = 0$ where $Q \neq 0$. This single charge BPS solution lifts to a solution of 10-dimensional type-IIB supergravity [30, 9] which admits 16 supersymmetries. By analysing the dipole moment of the five form $F^{(5)}$ the authors of [9] concluded that the 10-dimensional solution can be interpreted as the condensate of giant gravitons with angular momentum along ξ_1 and with a density of branes along α given by

$$dn = N \frac{Q}{L^2} \sin 2\alpha \, d\alpha. \tag{5.3}$$

The total number of giant gravitons is

$$n_{\text{tot}} = N \frac{Q}{L^2} \tag{5.4}$$

and the total angular momentum of the system is

$$P_{\xi_1} = \frac{N^2}{2} \frac{Q}{L^2}. \tag{5.5}$$

In terms of the giant graviton configurations counted in section 3, the density of giants in eq. (5.3) is given by $r_k = Q/L^2$ for all k in the continuum limit.[6]

This geometry was named the 'superstar' in [9]. This solution satisfies all our requirements except that it has an uncloaked null singularity. Nevertheless one may think of the 5-dimensional geometry as a black hole with a zero size horizon. On general grounds one expects that the Bekenstein-Hawking entropy formula for this geometry to get corrected in the quantum theory. As in the recent developments in the asymptotically flat 2-charge extremal black holes [16, 17] it is conceivable that one recovers the finite entropy for the superstar geometry too after quantum corrections. However this analysis is outside the scope of this paper. Instead what we do here is to follow the reasoning of [12, 18, 19] and place a stretched horizon in our geometry.

6. Stretched horizon and the entropy of superstar

Recall that the total R-charge of the single charge superstar is

$$Q_{total} = \frac{N^2 Q}{2L^2} \tag{6.1}$$

Substituting this into eq. (4.12) we get the entropy prediction for the 1/2-BPS black hole coming from both the CFT and string theory:

$$S_{BH} = \left(\frac{\pi}{\sqrt{3}}\right) N \sqrt{Q_0} \tag{6.2}$$

where we have defined $Q_0 = Q/L^2$. For large-N and for any finite value of Q_0 this entropy is macroscopic. Next we would like to reproduce this answer using the mechanism of stretched horizon for the single charge superstar geometry.

A stretched horizon in the context of heterotic string compactified down to four dimensions has been defined first in [12] as the place where the string world-sheet becomes strongly coupled. However this is the correct prescription if the leading corrections to the corresponding black hole geometry come from the α' corrections to supergravity as opposed to string loop g_s corrections. For black holes in AdS the two relevant expansion parameters are $1/N$ (or the 10-dimensional plank length measured in units of the AdS radius as $l_{pl}^8/L^8 \sim N$) and λ (or the α' measured in units of AdS radius as $L^4/\alpha'^2 \sim \lambda$). If the dominant correction to the geometry comes from plank length corrections then a D3-brane probe which sees the effects of finite N is more appropriate than a fundamental string. However, a priori it is not evident which type of corrections are dominant in our case.

A different point of view has recently been advocated by Mathur and collaborators (see [18, 19] for instance). Here the basic idea is that the brane system that makes up the black hole has a nonzero size. Further, there are some excitations of the brane system which makes up the black hole that can spread over long distances compared to the relevant fundamental length. One has to place a stretched horizon so that these fluctuations are inside it. Typically this length scale is set by the smallest possible excitation of the system. We shall use this argument below.

In our case each microstate of the superstar geometry can be thought of as a giant graviton configuration with fixed total R-charge. This system is also expected to have a finite size. The source of this finite size can be traced to the configuration of fermions in the phase space representation of [8, 6]. There, one may think of the system as an incompressible fluid. Any excited state requires creating holes in the fermion droplet and therefore the droplet spreads over a larger area. This spreading directly translates into a spread in the radial direction in the AdS space using the results of [6]. To estimate the extent of a given microstate let us note that the lightest excitation of the giant graviton system is to add a single unit of angular momentum. This can be done by adding a single giant graviton to any of the microstate configurations of giants at level-1 (or shift a giant graviton from level k to $k+1$). We have to convert this energy into a length scale. Roughly speaking if we use this energy to create a dual-giant graviton then the length scale that sets the size of this dual-giant, in this case given by $L/\sqrt{N}$, should also set the size of the stretched horizon.

From the point of view of the fermion system the smallest excitation is to excite the topmost fermion by one energy level.[7] It is easy to see from the analysis of [6] that to

_____

[7]This excitation can be thought of as creating a smallest size giant graviton or equivalently a smallest size dual-giant with a unit of R-charge as in section 3.

accomplish this operation it costs energy of order N and the disturbance of this excitation is $\delta R \sim L^2/N$ where $R = \mu_1 L^2 (1 + r^2/L^2)^{1/2}$ [6] with R being the radial coordinate in the phase space and $\mu_1 = \cos\alpha$. So we see that supergravity solutions of two configurations of giants which differ by this excitation roughly differ from each other within the range of $0 \leq r \leq c_0 L/\sqrt{N}$ where c_0 is a pure number.

Following the analogy of [18, 19] we postulate that the size of the stretched horizon is determined by the same length scale that sets the size of a dual-giant. Therefore we propose to place the stretched horizon at:

$$r_0 = \frac{c_0 L}{\sqrt{N}} \tag{6.3}$$

where c_0 is a pure numerical coefficient.

Substituting $q_1 = Q$ $\mu = q_2 = q_3 = 0$ into (5.1) the 5-dimensional metric of the single charge superstar reads

$$ds_5^2 = -H^{-2/3}\left(1 + \frac{r^2}{L^2}H\right)dt^2 + H^{1/3}\left[\frac{dr^2}{1 + \frac{r^2}{L^2}H} + r^2 d\Omega_3^2\right] \tag{6.4}$$

where $H = 1 + \frac{Q}{r^2}$. We assume that $Q_0 = Q/L^2 \ll 1$ so that the metric is asymptotically globally AdS_5. The metric on a space-like surface at the position $r = r_0$ is:

$$ds_{horizon}^3 = \left(1 + \frac{L^2 Q_0}{r_0^2}\right)^{1/3} r_0^2 d\Omega_3^2 \tag{6.5}$$

where $d\Omega_3^2$ is the metric on a unit 3-sphere S^3. The area of the stretched horizon becomes:

$$A \approx c_0^2 \, \text{Vol}_{S^3} \, \frac{L^3 Q_0^{1/2}}{N}. \tag{6.6}$$

The 5-dimensional Newton's constant is given by:

$$G_N = \frac{8\pi^3 g_s^2 \alpha'^4}{L^5} \sim \frac{L^3}{N^2} \tag{6.7}$$

Then using the Bekenstein-Hawking entropy formula we get:

$$S_{BH} \sim \frac{A_{sh}}{G_N} \sim c_0^2 \left(\frac{L^4}{N g_s \alpha'^2}\right)^2 N\sqrt{Q_0} \sim \mathcal{C}_0 \, N\sqrt{Q_0}. \tag{6.8}$$

Up to a numerical coefficient this precisely matches with the entropy prediction (6.2) of the gauge and string theory microstate counting.

Even though we relied on the D3-brane probe analysis to get the size of the stretched horizon in the superstar geometry, it is clear that if we demanded the entropy of 1/2-BPS states in eq.(33) to be carried by the superstar geometry and worked backwards to find the location of the stretched horizon, we would have ended up with the same answer as in eq.(34). So we can take this as an evidence for the appropriateness of the D3-brane probe as opposed to the fundamental string probe.

7. M-theory examples

Let us also consider similar singular geometries in the M-theory examples $AdS_4 \times S^7$ and $AdS_7 \times S^4$. An analysis on lines of [9] was carried out in [20, 21].

7.1 Superstar in $AdS_4 \times S^7$

The black hole in this case can carry four charges. We again restrict ourselves to the single charge extremal limit. The 4 dimensional solution is [31, 32]:

$$
\begin{aligned}
ds_4^2 &= -H^{1/2}f\,dt^2 + H^{1/2}(f^{-1}dr^2 + r^2 d\Omega_2^2), \\
X_1 &= H^{-3/4}, \qquad X_2 = X_3 = X_4 = H^{1/4}, \qquad A = (1 - H^{-1})\,dt
\end{aligned}
\tag{7.1}
$$

where

$$
f = 1 + \frac{r^2}{L_4^2}H, \qquad H = 1 + \frac{Q}{r}.
\tag{7.2}
$$

This solution also has a null singularity at $r = 0$ and admits an interpretation [20, 21] as a backreaction of a source of giant gravitons now made out of $M5$ branes wrapping some $S^5 \subset S^7$ and carrying angular momentum. We refer the reader to [20, 21] for the details of this analysis and be content here with a summary. The total number of giants is

$$
n = 2^{1/2}N^{1/2}\frac{Q}{L_4}
\tag{7.3}
$$

and the total angular momentum of the solution is:

$$
P_{\xi_1} = \frac{2N^{3/2}}{3\sqrt{2}}\frac{Q}{L_4}.
\tag{7.4}
$$

The 4-dimensional Newton's constant is:

$$
G_4 \sim \frac{L_4^2}{N^{3/2}}.
\tag{7.5}
$$

There exist dual-giants made of $M2$ branes wrapping the $S^2 \subset AdS_4$ and carrying angular momentum in S^7. The sizes of these are [4]:

$$
r = \frac{L_4}{\sqrt{N}}\frac{P_{\xi_1}}{2}.
\tag{7.6}
$$

Using the hypothesis that the stretched horizon is to be placed at a length scale decided by the size of the dual-giant we set:

$$
r_0 = \frac{c_0\,L_4}{\sqrt{N}}.
\tag{7.7}
$$

where c_0 is again a pure number. Using the metric above it is easy to calculate the area of this horizon and we find:

$$
A_{sh} \sim \frac{c_0 L_4^2}{N^{3/4}}\sqrt{Q_0}
\tag{7.8}
$$

where we have defined $Q_0 = Q/L_4$. This implies:

$$S_{bh}^{(4)} \sim \frac{A_{sh}}{G_4} \sim N^{3/4}\sqrt{Q_0}. \tag{7.9}$$

Even though there is no lagrangian description for the dual field theory to get a prediction for this entropy one can treat the chiral primaries again using the picture of fermions in a harmonic oscillator potential. One can also do a counting of giants or dual-giants on the lines of section 4.2 and the answer for the partition function comes out similar to eq. (4.12). Therefore the prediction from the CFT and the M-theory side for the entropy of our black hole is again:

$$S = \mathcal{C}\sqrt{\Delta} + \cdots \tag{7.10}$$

where $\cdots$ again mean corrections which are small in large-N and large-Δ limit. Substituting P_{ξ_1} from eq. (7.4) for Δ in eq. (7.10) gives precisely the same functional dependence of the entropy (7.9) on the charge Q_0 and N.

7.2 Superstar in $AdS_7 \times S^4$

There exists a similar null singularity with 16 supercharges in this background as well. The solution in 7 dimensions is [31, 32]:

$$ds_7^2 = -H^{-4/5}f\,dt^2 + H^{1/5}(f^{-1}dr^2 + r^2 d\Omega_5^2),$$
$$X_1 = H^{-3/5}, \qquad X_2 = H^{2/5}, \qquad A = (1 - H^{-1})\,dt \tag{7.11}$$

where

$$f = 1 + \frac{r^2}{L_7^2}H, \qquad H = 1 + \frac{Q}{r^4}. \tag{7.12}$$

The total charge of this solution is $P_{\xi_1} = (2N^3Q)/(3L_7^4) \sim N^3Q_0$ where $Q_0 = Q/L_7^4$. The size of a dual-giant is $r \sim P_{\xi_1}^{1/4}(L_7/\sqrt{N})$ and the 7-dimensional Newton's constant is $G_7 \sim L_7^5/N^3$. Using the same hypothesis that the position of the stretched horizon is determined by the length scale that determines the size of the dual-giant $L_7/\sqrt{N}$, we again take $r_0 = c_0 \frac{L_7}{\sqrt{N}}$ to be the horizon radius. Then the entropy works out to be:

$$S_{bh}^{(7)} \sim N^{3/2}\sqrt{Q_0} \tag{7.13}$$

which again matches (up to non-vanishing numerical factors) with the prediction coming from counting the corresponding giant configurations with the total angular momentum $P_{\xi_1} = (2N^3Q)/(3L_7^4) \sim N^3Q_0$. Thus we conclude that the null singularities of M-theory should also be genuine black holes at the quantum level.

8. More consequences

Let us state some of the consequences of the duality considered in section 3. Suppose we start with the configuration of all N fermions being in their ground state. This represents the vacuum state of the dual geometry, namely, empty $AdS_5 \times S^5$ [8, 6]. Now consider

a generic fermion excitation. This is represented by AdS/CFT *either* by a configuration of dual-giants *or* by a configuration of giants. But both correspond to describing the same fermion system either in terms of particles or in terms of holes both of which should be equivalent. This leads to the statement that a given giant configuration is equivalent to a corresponding dual-giant configuration. However similar to the fermion system the corresponding supergravity solutions would also have to be identical.

Another consequence is the following. Again start with a giant graviton configuration labeled by $\vec{b}_1 = (r_1, r_2, \ldots, r_N)$. Now consider adding a dual-giant to this system. Since giants are holes in the fermion picture the topmost fermion is away from its ground state position $N - 1$ and this distance is given by the number of holes. As a consequence to excite the kth fermion we have to have

$$s_k > \sum_{i=k}^{N} r_i. \tag{8.1}$$

For $k = 1$ this implies that a dual-giant will have a non-zero radius only when its angular momentum s_1 exceeds the total number $\sum_{i=1}^{N} r_i$ of the giant gravitons. Let us test this prediction in a simple context.

To test this prediction we generically need the backreacted geometry of the giant configuration under consideration. Below we work with the single charge superstar which contains a total of NQ_0 D3-branes. So if we try to place a dual-giant D3-brane as a probe in this background it should not have a non-zero radius unless the angular momentum of the dual-giant exceeds NQ_0.

Fortunately the probe brane analysis in question has already been studied in [9]. We summarise the results here. For a given angular momentum P_{ξ_i} in S^5 the equations of motion for the world-volume theory are satisfied if the radius r and the angles α, β satisfy the following relations:

$$\frac{r^2}{L^2} = \sum_{i=1}^{3} \left(\frac{P_{\xi_i}}{N} - \frac{q_i \mu_i^2}{L^2} \right),$$
$$\mu_i^2 = \frac{\frac{P_{\xi_i}}{N} - \frac{q_i \mu_i^2}{L^2}}{\sum_{j=1}^{3} \left(\frac{P_{\xi_j}}{N} - \frac{q_j \mu_j^2}{L^2} \right)}. \tag{8.2}$$

Let us specialise to $P_{\xi_2} = P_{\xi_3} = 0$ and $q_2 = q_3 = 0$. Then the second of the equations (8.2) can be solved if $\mu_1 = 1$ and $\mu_2 = \mu_3 = 0$. That is $\alpha = \pi/2$. Substituting these into the first equation gives:

$$\frac{r^2}{L^2} = \frac{P_{\xi_1}}{N} - \frac{q_1}{L^2} \tag{8.3}$$

That is, the typical radius at which the dual-giant settles down is:

$$r = \frac{L}{\sqrt{N}} \left[P_{\xi_1} - \frac{N q_1}{L^2} \right]^{1/2} = \frac{L}{\sqrt{N}} \left[P_{\xi_1} - N Q_0 \right] \tag{8.4}$$

Thus we see that the dual-giant will have non-vanishing radius only when its angular momentum exceeds the total number of giants NQ_0 as predicted. Similar predictions

apply for the M-theory cases as well and one can verify them using the probe brane analyses of [21]. One should be able to verify this prediction further by probing various supergravity solutions presented in [6] with D3-brane configurations with angular momenta. We will not do this here however.

It follows from section 3 that M giant gravitons with angular momentum N are dual to N dual-giants with M units of angular momentum each. However depending on how big the values of M and N are relative to each other it should be possible to think of either M or N as the number of giants/dual-giants in the supergravity background of the other. A similar 'correspondence principle' was proposed in [20].

9. Conclusion

In this paper we reconsidered the 1/2-BPS configurations on both the gauge theory and string theory sides of the AdS/CFT correspondence. We have argued that there is an upper limit on the number of dual-giants one can place in $AdS_5 \times S^5$. Using the fermion picture that arises from the matrix model truncation of the gauge theory we have set up a mapping between the configurations of fermions, giants and the dual-giants. These important maps provide two different ways of bosonising the fermionic quantum mechanical system.[8] This leads to a duality map between giant graviton configurations and the dual-giant graviton configurations which is hinted at by [28] in the M-theory context. As has been discussed in the text this duality is the direct consequence of particle-hole duality of the fermion description of the 1/2 BPS spectrum of the CFT. From the bulk point of view this is a duality between various semi-classical D3-brane configurations and is important in a avoiding over counting the 1/2-BPS states in the bulk. Some other consequences of the mapping proposed have also been tested in simple cases in section 8.

Further, we found the partition function of the 1/2-BPS states both by counting the chiral primaries using the matrix model and various giant graviton configurations which match. The density of 1/2-BPS states is shown to grow exponentially with the total R-charge. Then we proposed that this density of states should be carried by the single charge 'superstar' geometry of [9]. A stretched horizon was proposed and shown to reproduce the entropy predicted by both the CFT and the string theory microstate counting. Since the size of the minimal dual-giant probe gave the correct location of the stretched horizon it suggests that the plank length (or $1/N$) corrections are more dominant than the α' corrections. It is possible that the α' corrections could be relevant for the subleading corrections to the entropy formula.

One might ask why the superstar geometry should be the relevant classical limit of the R-charged supersymmetric black hole. For this let us reiterate that the superstar geometry is the only known 1/2-BPS geometry which is a solution to a 5-dimensional (gauged) supergravity and it is the geometry that arises as the extremal limit of a non-extremal 5-dimensional R-charged black hole. More importantly this is emerges as the limiting geometry of the supersymmetric black hole solutions with finite size horizons constructed

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[8]This bosonization has been proved at the level of operators acting on the corresponding quantum mechanical hilbert spaces recently in [39].

in [34, 35] when the extra charges in their solutions are turned off. It will however be interesting if one can argue more concretely for the conjecture that the superstar carries the entropy coming from the degeneracy of chiral primaries on the lines of [16, 17]. This will require Wald's formula [13] for entropy applied to gauged supergravities.

In recent times a new understanding of the notion of stretched horizon has been provided by Mathur *et al* (see for instance [18]). It will be interesting to see if a similar picture to the D1-D5 system emerges even for the geometries considered here. In particular, one should be able to extract the relevant microstate geometries from those found recently in [6]. One must then be able to 'coarse-grain' over these geometries and get the entropy formula. See [33] for a related discussion.

The partition function of 1/2-BPS states in the $N \to \infty$ limit matches with that of the 2-dimensional CFT of a single chiral boson (see for instance [40, 25]). It would be interesting to explore this relation further (see also [37]).[9]

A class of supersymmetric black holes in AdS_5 with finite area horizons of spherical topology have been discovered recently [34, 35]. When lifted to 10 dimensions they preserve just two supersymmetries [36]. Of course understanding their entropies from the CFT side remains an important outstanding problem. See [37] for a discussion on the microstate counting of the near extremal R-charge black holes in AdS.

Acknowledgments

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